Mark Scheme (Results)

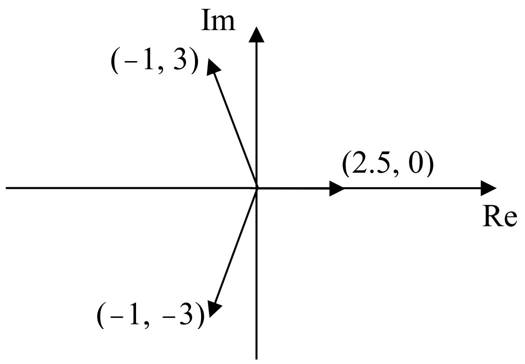
October 2021

Pearson Edexcel International A Level
In Further Pure Mathematics F1 (WFM01)
Paper 01

Question Number	Scheme	Notes	Marks
1(a)	$\mathbf{A}^{-1}=\dfrac{1}{3\times-2-a\times-2}\begin{pmatrix}-2 & -a\\ 2 & 3\end{pmatrix}$	Complete method for the inverse. Allow slips in the determinant and at most one error in the adjoint matrix.	M1
	$=\dfrac{1}{2a-6}\begin{pmatrix}-2 & -a\\ 2 & 3\end{pmatrix}$	Correct inverse	A1
			(2)
(b)	$\mathbf{A}+\mathbf{A}^{-1}=\mathbf{I}\Rightarrow\begin{pmatrix}3 & a\\ -2 & -2\end{pmatrix}+\dfrac{1}{2a-6}\begin{pmatrix}-2 & -a\\ 2 & 3\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	Sets up the correct matrix equation with their $\mathbf{A}^{-1}$. The identity may be left as **I** as long at least one equation is processed correctly and no equation implies an incorrect identity matrix.	M1
	$3+\dfrac{2}{6-2a}=1,\ a+\dfrac{a}{6-2a}=0,\ -2+\dfrac{2}{2a-6}=0,\ -2+\dfrac{3}{2a-6}=1\Rightarrow a=...$ Uses one of the elements to set up a suitable equation and solves for *a*. Allow a sign slip in the $6-2a$ but have correct coefficient of **I**		dM1
	$a=\dfrac{7}{2}$ oe	Correct value and no others	A1
			(3)
			Total 5

Question Number	Scheme	Notes	Marks
2(a)	$f(x) = 7\sqrt{x} - \frac{1}{2}x^3 - \frac{5}{3x} \qquad x > 0$		
	$f(2.8) = 0.1420022...$ $f(2.9) = -0.8486421...$	Attempts both f(2.8) and f(2.9) with at least one correct to 1 s.f.	M1
	Sign change (positive, negative) and $f(x)$ is **continuous** therefore (a **root**) α is between $x = 2.8$ and $x = 2.9$	Both f(2.8) = awrt 0.1 (or truncated) and f(2.9) = awrt –0.8 (or truncated), sign change, continuous and minimal conclusion.	A1
			(2)
(b)(i)	$f'(x) = \frac{7}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^2 + \frac{5}{3x^2}$	$x^n \to x^{n-1}$ at least once	M1
		Correct derivative	A1
(b)(ii)	$x_1 = 2.8 - \frac{f(2.8)}{f'(2.8)} = 2.8 - \frac{0.142002276...}{-9.4557649...}$	Correct application of Newton-Raphson. If no substitution/values see accept a correct statement followed by a value for the attempt.	M1
	$= 2.815$	cao following a correct derivative.	A1
			(4)
(c)	$\frac{2.9 - \alpha}{0.8486421875...} = \frac{\alpha - 2.8}{0.1420022762...}$	Any correct or implied linear interpolation statement.	B1
	$\alpha = \frac{2.8 \times 0.8486421875... + 2.9 \times 0.1420022762...}{0.8486421875... + 0.1420022762...} = ...$ Rearranges an equation suitable form (e.g. allow sign errors in interpolation statement) to give $\alpha = \ldots$		M1
	$= 2.814$	cao	A1
			(3)
Alt (c)	$\frac{x}{0.1420022762...} = \frac{0.1}{0.1420022762... + 0.8486421875...}$	Any correct or implied linear interpolation statement for x distance.	B1
	$\alpha = 2.8 + x = 2.8 + \frac{0.4 \times 0.1420022762...}{0.8486421875... + 0.1420022762...} = ...$ Rearranges and adds 2.8 to give $\alpha = \ldots$		M1
	$= 2.814$	cao	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
3	$2x^2 - 5x + 7 = 0$		
(a)	$\alpha + \beta = \frac{5}{2}, \quad \alpha\beta = \frac{7}{2}$	Both	B1
			(1)
(b)(i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	Attempts to use a correct identity	M1
	$= \left(\frac{5}{2}\right)^2 - 2\left(\frac{7}{2}\right) = -\frac{3}{4}$	cso – must have scored the B1	A1
(ii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$	Attempts to use a correct identity	M1
	$= \left(\frac{5}{2}\right)^3 - 3\left(\frac{7}{2}\right)\left(\frac{5}{2}\right) = -\frac{85}{8}$	cso – must have scored the B1	A1
			(4)
(c)	$\text{Sum} = \frac{1}{\alpha^2 + \beta} + \frac{1}{\beta^2 + \alpha} = \frac{\alpha^2 + \beta + \beta^2 + \alpha}{(\alpha^2 + \beta)(\beta^2 + \alpha)}$ $= \frac{\alpha^2 + \beta^2 + \alpha + \beta}{\alpha^2\beta^2 + \alpha^3 + \beta^3 + \alpha\beta} = \frac{-\frac{3}{4} + \frac{5}{2}}{\frac{49}{4} - \frac{85}{8} + \frac{7}{2}} \left(= \frac{14}{41}\right)$ Attempts sum – substitutes their into a correct numerator must but allow slips in the denominator as long as 4 terms are produced from the expansion.		M1
	$\text{Product} = \frac{1}{\alpha^2 + \beta} \times \frac{1}{\beta^2 + \alpha} = \frac{1}{\alpha^2\beta^2 + \alpha^3 + \beta^3 + \alpha\beta} = \frac{1}{\frac{49}{4} - \frac{85}{8} + \frac{7}{2}} \left(= \frac{8}{41}\right)$ Attempts product – must be correct expansion of denominator with their values.		M1
	$x^2 - \frac{14}{41}x + \frac{8}{41} (= 0)$	Applies $x^2 -$ (their sum)x + their product (= 0) Depends on at least one previous M awarded.	dM1
	$41x^2 - 14x + 8 = 0$	Allow any integer multiple. Must include "=0"	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
4	$\mathrm{f}(z)=2z^3-z^2+az+b$		
(a)	$(z=)-1+3\mathrm{i}$	Correct complex number	B1
			(1)
(b)	$z=-1\pm3\mathrm{i}\Rightarrow\big(z-(-1+3\mathrm{i})\big)\big(z-(-1-3\mathrm{i})\big)\to z^2+\ldots z+\ldots$ Or e.g. Sum = –2, Product = $(-1)^2-(3\mathrm{i})^2=10\to z^2+\ldots z+\ldots$ Correct strategy to find the quadratic factor		M1
	$z^2+2z+10$	Correct expression	A1
	$\mathrm{f}(z)=(z^2+2z+10)(2z-5)$	Uses an appropriate method to find the linear factor	M1
	$\Rightarrow \mathrm{f}(z)=2z^3-z^2+10z-50$ or $a=10,\quad b=-50$	Correct cubic or correct constants	A1
			(4)
(c)		$-1\pm3\mathrm{i}$ correctly plotted with vectors or dots or crosses etc. May be labelled by coordinates or on axes. Do not be concerned about scale but should look like reflections in the real line.	B1
		(2.5, 0) plotted correctly or follow through their non-zero real root correctly plotted. May be labelled by coordinates or on axes. Do not be too concerned about scale but e.g (2.5,0) should be further from *O* than (−1,0) is.	B1ft
			(2)
			Total 7
Alt (b)	$\mathrm{f}(-1+3\mathrm{i})=0\Rightarrow 2(-1+3\mathrm{i})^3-(-1+3\mathrm{i})^2+a(-1+3\mathrm{i})+b=0$ $\mathrm{Im}\big(\mathrm{f}(-1+3\mathrm{i})\big)=0\Rightarrow 2(9-27)-(-6)+3a=0\Rightarrow a=\ldots$ Or e.g. $\mathrm{f}(-1+3\mathrm{i})-\mathrm{f}(-1-3\mathrm{i})=0\Rightarrow 2\big(2(9\mathrm{i}-27\mathrm{i})-(-6\mathrm{i})+3a\mathrm{i}\big)=0\Rightarrow a=\ldots$ Correct full strategy to find one constant.		M1
	$a=10$ or $b=-50$	One correct value.	A1
	E.g. $\mathrm{Re}(\mathrm{f}(-1+3\mathrm{i}))=0\Rightarrow 2(-1+27)-(1-9)-a+b=0\Rightarrow b=\ldots$ Correct method to find the second constant.		M1
	$a=10$ and $b=-50$ or $\mathrm{f}(z)=2z^3-z^2+10z-50$	Correct constants or correct cubic	A1
			(4)

Question Number	Scheme	Notes	Marks
5(a)	$r(r-1)(r-3)=r^3-4r^2+3r$	Correct expansion	B1
	$\sum_{r=1}^{n}(r^3-4r^2+3r)=\frac{1}{4}n^2(n+1)^2-4\frac{1}{6}n(n+1)(2n+1)+3\times\frac{1}{2}n(n+1)$ M1: Attempt to use at least two of the standard formulae correctly A1: Correct expression		M1A1
	$=\frac{1}{12}n(n+1)[3n(n+1)-8(2n+1)+18]$	Attempt to factorise $\frac{1}{12}n(n+1)$ from an expression with these factors. Depends on previous M.	dM1
	Note: for attempts that first expand to a quartic this mark may be awarded at the point the relevant factors are taken out provided a suitable quadratic factor is seen before the final answer.		
	$=\frac{1}{12}n(n+1)[3n^2-13n+10]$ $=\frac{1}{12}n(n+1)(n-1)(3n-10)$*	Cso with $3n^2-13n+10$ (or another appropriate correct quadratic) seen before the final printed answer.	A1*
			(5)
(b)	$\sum_{r=n+1}^{2n+1}r(r-1)(r-3)=\frac{1}{12}(2n+1)(2n+2)2n(6n-7)-\frac{1}{12}n(n+1)(n-1)(3n-10)$ Attempts f (2n + 1) – f (n)		M1
	$=\frac{1}{12}n(n+1)[4(2n+1)(6n-7)-(n-1)(3n-10)]=\frac{1}{12}n(n+1)(...n^2+...n+...)$ Attempt to factor out $\frac{1}{12}n(n+1)$ and simplify the rest to 3 term quadratic expression. For attempts expanding to a quartic first, score for reaching an expression of the correct form.		dM1
	$=\frac{1}{12}n(n+1)(45n^2-19n-38)$	Cao	A1
			(3)
			Total 8

Question Number	Scheme	Notes	Marks
6(a)	$\left(\dfrac{a}{\sqrt{k}},\ a\sqrt{k}\right)$	Correct coordinates – need not be simpified, so accept any equivalents.	B1
	$xy = a^2 \Rightarrow y = a^2x^{-1}$ $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = -a^2x^{-2} = -a^2\left(\dfrac{a}{\sqrt{k}}\right)^{-2} \ (= -k)$	Correct method for the gradient of the tangent at P. Must have substituted for x (and y) in their derivative.	M1
	$y - a\sqrt{k} = -k\left(x - \dfrac{a}{\sqrt{k}}\right)$ oe or $y = -kx + c \Rightarrow c = a\sqrt{k} + \dfrac{ka}{\sqrt{k}}$; $\Rightarrow y = -kx + 2a\sqrt{k}$ oe	M1:Correct straight line method for the tangent at P A1: Correct equation. Need not be fully simplified but do not accept $\sqrt{a^2}$ terms left unsimplified. ISW after a suitable correct equation seen.	M1A1
			(4)
(b)	$x = 0 \Rightarrow y = \ldots \quad y = 0 \Rightarrow x = \ldots$	Uses $x = 0$ **and** $y = 0$ to find A **and** B	M1
	$A\left(\dfrac{2a}{\sqrt{k}},\ 0\right) \quad B\left(0, 2a\sqrt{k}\right)$	Correct coordinates with same criteria as in (a).	A1
			(2)
(c)	Area $= \dfrac{1}{2} \times 2a\sqrt{k} \times \dfrac{2a}{\sqrt{k}} = \ldots$	Fully correct strategy for the area	M1
	$= 2a^2$ Which is independent of k	All correct with conclusion	A1
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
7(i)(a)	$\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Correct matrix	B1
			(1)
(b)	$\begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$	Correct matrix	B1
			(1)
(c)	$\begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Attempt to multiply the right way round. Implied by a correct answer (for their (a) and (b)) if no working is shown, but M0 if incorrect with no working.	M1
	$\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{5}{2} & -\frac{5\sqrt{3}}{2} \end{pmatrix}$	Correct matrix	A1
			(2)
(ii)(a)	$\begin{vmatrix} k & k+3 \\ -5 & 1-k \end{vmatrix} = k(1-k)-(-5)(k+3)$	Correct method for the determinant. (Allow miscopy slips only. So $k(1-k)-5(k+3)$ is M0 without further evidence.)	M1
	$=-k^2+6k+15$	Correct simplified expression	A1
			(2)
(b)	$-k^2+6k+15=\frac{16k}{2} \Rightarrow k=...$ or $-k^2+6k+15=-\frac{16k}{2} \Rightarrow k=...$	Correct strategy for establishing at least one value for k	M1
	One of $k=-5,\ 3,\ -1,\ 15$	Any one correct value. Note that the negative values may be rejected here.	A1
	$k=3$ **and** $k=15$ or $k=-5,\ 3$ **and** $k=-1,\ 15$	Both correct positive values and no others. Condone the inclusion of the negative values if given.	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
8(a)	$y^2 = 20x \Rightarrow y = \sqrt{20}x^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{20}}{2}x^{-\frac{1}{2}} = \frac{\sqrt{20}}{2\sqrt{5p^2}}$ or $y^2 = 20x \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 20 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{10}{y} = \frac{10}{10p}$ or $x = 5p^2, y = 10p \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{10}{10p}$	Correct strategy for finding $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of p	M1
	$y - 10p = \frac{1}{p}\left(x - 5p^2\right)$ or $y = \frac{1}{p}x + c \Rightarrow c = 10p - \frac{1}{p} \times 5p^2$	Correct straight line method	M1
	$py - x = 5p^2$ *	Cso	A1*
			(3)
(b)	$(0, 5p)$	Correct coordinates	B1
			(1)
(c)	$(5, 0)$	Correct coordinates	B1
			(1)
(d)	$y = \frac{2}{p}x$	Correct equation for l_2	B1
	E.g. $y = -\frac{5p}{5}(x - 5)$ or $y = -px + c \rightarrow c = 5p$	Correct strategy for the equation of l_1 (providing it has non-zero gradient)	M1
	$y = \frac{2}{p}x \Rightarrow p = \frac{2x}{y} \Rightarrow y = -\frac{2x}{y}(x - 5)$	Eliminates p to obtain an equation connecting x and y	M1
		Correct equation in any form	A1
	$2x^2 + y^2 = 10x$*	Fully correct proof	A1*
			(5)
	Alternative for last 3 marks of (d)		
	$y = \frac{2}{p}x,\ y = -\frac{5p}{5}(x - 5)$ $\Rightarrow x = \ldots, y = \ldots$	Solves simultaneously to find x and y in terms of p	M1
	$x = \frac{5p^2}{p^2 + 2},\ y = \frac{10p}{p^2 + 2}$	Correct coordinates for B	A1
	$2x^2 + y^2 = 2\left(\frac{25p^4}{\left(p^2+2\right)^2}\right) + \frac{100p^2}{\left(p^2+2\right)^2} = \frac{50p^4 + 100p^2}{\left(p^2+2\right)^2} = \frac{50p^2\left(p^2+2\right)}{\left(p^2+2\right)^2} = 10x$* Completes the proof by substituting into the given equation and shows sufficient working to establish the equivalence (as above)		A1*
			Total 10

Question Number	Scheme	Notes	Marks
9(i)	$n=1 \Rightarrow u_1 = 3\times2-2\times3=0$ $n=2 \Rightarrow u_2 = 3\times2^2-2\times3^2=-6$	Shows the result is true for n = 1 **and** n = 2. Ignore references to n = 3.	B1
	Substitutes $u_k = 3\times2^k-2\times3^k$ and $u_{k+1}=3\times2^{k+1}-2\times3^{k+1}$ into $(u_{k+2}=)5u_{k+1}-6u_k = 5\left(3\times2^{k+1}-2\times3^{k+1}\right)-6\left(3\times2^k-2\times3^k\right)$ (The inductive assumption may be tacit for this mark.)		M1
	$(u_{k+2})=15\times2^{k+1}-10\times3^{k+1}-18\times2^k+12\times3^k$ $=15\times2^{k+1}-9\times2^{k+1}-10\times3^{k+1}+4\times3^{k+1}$ $=6\times2^{k+1}-6\times3^{k+1}$	Gathers to a correct two term expression. Accept alternative forms such as $12\times2^k-18\times3^k$	A1
	$u_{k+2}=3\times2^{k+2}-2\times3^{k+2}$	Achieves this result with no errors – must be clear it is u_{k+2} but this may have been seen at the start.	A1
	If the result is **true for $n = k$ and $n = k + 1$ then** it is **true for $n = k + 2$**. As the result has been shown to be **true for $n = 1$ and $n = 2$** then the result is **true for all n**.		A1cso
	Correct conclusion including all the bold points in some form. Depends on all previous marks.		
			(5)
(ii)	$\mathrm{f}(n)=3^{3n-2}+2^{4n-1}$		
	$\mathrm{f}(1)=3^1+2^3=11$	Shows the result is true for n = 1	B1
	$\mathrm{f}(k+1)-\mathrm{f}(k)=3^{3(k+1)-2}+2^{4(k+1)-1}-3^{3k-2}-2^{4k-1}$ $=27\times3^{3k-2}+16\times2^{4k-1}-3^{3k-2}-2^{4k-1}=26\times3^{3k-2}+15\times2^{4k-1}\left(=\frac{26}{9}3^{3k}+\frac{15}{2}2^{4k}\right)$ Attempts f (k + 1) – f (k) and reaches $\alpha\times3^{3k-2}+\beta\times2^{4k-1}$ or $\alpha\times3^{3k}+\beta\times2^{4k}$ (Mark variations on the theme as appropriate here and in the following marks.)		M1
	$=15\times\left(3^{3k-2}+2^{4k-1}\right)+11\times3^{3k-2}$ **or** $=26\times\left(3^{3k-2}+2^{4k-1}\right)-11\times2^{4k-1}$	Correct expression with f(k) evident.	A1
	$\mathrm{f}(k+1)=16\mathrm{f}(k)+11\times3^{3k-2}$ **or** $\mathrm{f}(k+1)=27\mathrm{f}(k)-11\times2^{4k-1}$	Makes f(k + 1) the subject and states divisible by 11 (oe – may be implied by conclusion), or gives full reason why f(k + 1) is divisible by 11. **Dependent on first M**	dM1
	If the result is **true for $n = k$ then** it **is true for $n = k + 1$**. As the result has been shown to be **true for $n = 1$**, then the result is **true for all n.**		A1cso
	Correct conclusion including all the bold points in some form. Depends on all previous marks.		
			(5)
			Total 10

ALT 1	$f(1)=3^1+2^3=11$	Shows the result is true for $n=1$	B1
	$f(k+1)=3^{3k+1}+2^{4k+3}$ $f(k+1)=27\times 3^{3k-2}+16\times 2^{4k-1}$ or $3\times 3^{3k}+8\times 2^{4k}$ Attempts f $(k+1)$ and reduces power to $\alpha\times 3^{3k-2}+\beta\times 2^{4k-1}$ or $\alpha\times 3^{3k}+\beta\times 2^{4k}$		M1
	$f(k+1)=16\times\left(3^{3k-2}+2^{4k-1}\right)+11\times 3^{3k-2}$ or $f(k+1)=27\times\left(3^{3k-2}+2^{4k-1}\right)-11\times 2^{4k-1}$	Correct expression	A1
	$f\left(k+1\right)=16f\left(k\right)+11\times 3^{3k-2}$ **or** $f\left(k+1\right)=27f\left(k\right)-11\times 2^{4k-1}$	States divisible by 11 (oe– may be implied by conclusion) **Depends on first M**	dM1
	If the result is **true for *n* = *k* then** it **is true for *n* = *k* + 1**. As the result has been shown to be **true for *n* = 1**, then the result is **true for all *n*.**		A1cso
	Correct conclusion including all the bold points in some form. Depends on all previous marks.		
ALT 2	$f(1)=3^1+2^3=11$	Shows the result is true for $n=1$	B1
	Let $3^{3k-2}+2^{4k-1}=11M$		
	$f(k+1)=3^{3k+1}+2^{4k+3}$ $f(k+1)=27\left(11M-2^{4k-1}\right)+2^{4k+3}$ or $3^{3k+1}+16\left(11M-3^{3k-2}\right)$ Attempt f $(k+1)$ and expresses in terms of M		M1
	$f(k+1)=297M-11\times 2^{4k-1}$ or $176M+11\times 3^{3k-2}$	Correct expression	A1
	$f(k+1)=11\left(27M-2^{4k-1}\right)$ or $11\left(16M+3^{3k-2}\right)$	Takes out a factor of 11, or gives full reason why f(k+1) is divisible by 11. **Depends on first M**	dM1
	If the result is **true for *n* = *k* then** it **is true for *n* = *k* + 1**. As the result has been shown to be **true for *n* = 1**, then the result is **true for all *n*.**		A1cso
	Correct conclusion including all the bold points in some form. Depends on all previous marks.		
ALT 3	$f(1)=3^1+2^3=11$	Shows the result is true for $n=1$	B1
	$f(k+1)-\alpha f(k)=3^{3k+1}+2^{4k+3}-\alpha\left(3^{3k-2}+2^{4k-1}\right)$ Attempts f $(k+1)-\alpha$ f (k) where $\alpha=16$ or 27 or other appropriate value.		M1
	$=\left(27-\alpha\right)3^{3k-2}+\left(16-\alpha\right)2^{4k-1}$ **e.g.** $=11\times 3^{3k-2}\left(+(16-16)\times 2^{4k-1}\right)$ **or** $=\left((27-27)\times 3^{3k-2}\right)-11\times 2^{4k-1}$	Correct expression for their α where a common factor of 11 is clear. (E.g. $\alpha=5$)	A1
	E.g. $f\left(k+1\right)=16f\left(k\right)+11\times 3^{3k-2}$ **or** $f\left(k+1\right)=27f\left(k\right)-11\times 2^{4k-1}$	Makes f(k + 1) the subject in an expression where 11 is a clear common factor and states divisible by 11 (oe – may be implied by conclusion). **Dependent on first M**	dM1
	If the result is **true for *n* = *k* then** it **is true for *n* = *k* + 1**. As the result has been shown to be **true for *n* = 1**, then the result is **true for all *n*.**		A1cso
	Correct conclusion including all the bold points in some form. Depends on all previous marks.		